Generalized Rank Mapped Transmuted Distributions with Properties and Application: A Review

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Authors' contributions

This work was carried out in collaboration among all authors. All authors contributed immensely to the development of the article in all stages of the article formation. Author I designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author BD managed the literature searches and author SC managed the analyses of the study. All authors read and approved the final manuscript.

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Abstract

Generalizing probability distributions is a very common practice in the theory of statistics. Researchers have proposed several generalized classes of distributions which are very flexible and convenient to study various statistical properties of the distribution and its ability to fit the real-life data. Several methods are available in the literature to generalize new family of distributions. The Quadratic Rank Transmutation Map (QRTM) is a tool for the construction of new families of non-Gaussian distributions and to modulate a given base distribution for modifying the moments like the skewness and kurtosis with the ability to explore its tail properties and improve the adequacy of the distribution. Recently, a new family of transmutation map, defined as Cubic Rank Transmutation (CRT) has been used by several authors to develop new distributions with application to real-life data. In this article, we have done a review work on the existing generalized rank mapped transmuted probability distributions, available in the literature with various statistical properties such as the reliability, hazard rate and cumulative hazard functions, moments, mean, variance, moment-generating function, order statistics, generalized entropy and quantile function along with its applications. Some future works have also been discussed for generalized rank mapped transmuted distributions.

*Corresponding author: Email: bhanitadas83@gmail.com;
Keywords: Transmutation, Generalized Rank, Probability Distribution, Quadratic Transmutation, Cubic Rank Transmutation, Quartic Rank Transmutation.

1 Introduction

In many applied sciences such as medicine, engineering and finance, amongst others, modeling and analysing lifetime data are crucial. Several lifetime distributions have been used to model such kinds of data. The application of a statistical tool depends upon the underlying probability model of the data. As a result, huge numbers of probability distributions are being developed by numerous authors. However, there still remains large numbers of practical problems where the real data does not follow any of the classical or standard probability distributions. In the last two decades, several generalization approaches were adopted and practised which have received increased attention. Shaw and Buckley [1], developed an interesting method called the Quadratic Rank Transmutation Map (QRTM), which consists of introducing skewness or kurtosis in a symmetric or other (asymmetrical) distribution. Moreover, in order to capture the complexity of the data and increases the flexibility, new classes of Cubic Rank Transmutation (CRT) have been developed by Granzotto et al., [2]. Using this concept of CRT, various authors have proposed several cubic transmuted probability distributions which show better flexibility to handle more complex (bi-modal) data over quadratic transmuted distributions.

2 Developments in Transmuted Distributions

According to Shaw and Buckley [1], the cumulative distribution function (cdf) of the QRTM has the following simple quadratic form as

\[ F(x) = (1 + \lambda) \tilde{G}(x) - \lambda G(x)^2, \]  

where \( \epsilon \in \mathbb{R}, \lambda \in [-1, 1] \), \( G(x) \) is the cdf of the baseline distribution.

2.1 Quadratic rank transmutation map

Aryal and Tsokos [3] first emphasized on the technique given in (1) and introduced Transmuted Extreme Value distributions that would provide more distributional flexibility in reliability analysis. Further, various authors have developed several probability distributions using the QRTM given in (1) for various choices of baseline cdf \( G(x) \). Tahir and Cordeiro [4] and Rahman et al., [5] have provided a list for various quadratic transmuted distributions. At present, transmuted distributions are very common in the literature. An updated list of popular transmuted-G classes of distributions with its applications is given in Table 1.

Moreover, according to Granzotto et al., [2], the construction of the QRTM is simple and intuitive. Let \( X_1 \) and \( X_2 \) be independent and identically distributed random variables with distribution \( G(x) \). Then, consider

\[ Y \equiv \min(X_1, X_2), \quad \text{with probability } \pi, \]

\[ Y \equiv \max(X_1, X_2), \quad \text{with probability } 1 - \pi, \]

where \( 0 \leq \pi \leq 1 \). The distribution of \( Y \) is evidently

\[ F_Y(x) = \pi \Pr(\min(X_1, X_2) \leq x) + (1 - \pi) \Pr(\max(X_1, X_2) \leq x) \]

where \( \Pr \) is the probability of an event.

We know that \( F_{\min}(x) = 1 - [1 - G(x)]^n \) and \( F_{\max}(x) = [G(x)]^n \)

\[ F_Y(x) = \pi[1 - (1 - G(x))^2] + (1 - \pi)G^2(x) = 2\pi G(x) + (1 - 2\pi)G^2(x) \]

If we take \( 2\pi = \lambda \), the distribution is the well-known QRTM [2].

\[ F(x) = \lambda G(x) + (1 - \lambda)G(x)^2 \]  

(2)
Observe that at $\lambda = 1$ in (2), the above distribution gives the baseline distribution.

**Table 1. Development in quadratic rank transmuted distributions**

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Author(s) (Year)</th>
<th>Distribution</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Merovci [8]</td>
<td>Transmuted Exponentiated Exponential</td>
<td>Strength of 1.5cm glass fibres (England)</td>
</tr>
<tr>
<td>6</td>
<td>Ashour and Eltehiwy [10]</td>
<td>Transmuted Lomax</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>8</td>
<td>Elbatal and Elgarhy [12]</td>
<td>Transmuted Quasi-Lindley</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>10</td>
<td>Elbatal [14]</td>
<td>Transmuted Generalized Inverted Exponential</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>12</td>
<td>Ashour and Eltehiwy [26]</td>
<td>Transmuted Exponentiated Modified Weibull</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>14</td>
<td>Merovci et al., [18]</td>
<td>Transmuted Generalized Inverse Weibull</td>
<td>Failure times of 50 items</td>
</tr>
<tr>
<td>15</td>
<td>Merovci and Elbatal [19]</td>
<td>Transmuted Lindley-Geometric</td>
<td>Waiting times before services of 100 bank customers</td>
</tr>
<tr>
<td>18</td>
<td>Merovci and Puka [22]</td>
<td>Transmuted Pareto</td>
<td>Exceedances of flood peaks of the Wheaton River (Canada)</td>
</tr>
<tr>
<td>Sl. No.</td>
<td>Author(s) (Year)</td>
<td>Distribution</td>
<td>Applications</td>
</tr>
<tr>
<td>---------</td>
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<td>--------------</td>
</tr>
<tr>
<td>19</td>
<td>Iriarte and Astorga [23]</td>
<td>Transmuted Maxwell</td>
<td>Energy consumption during a certain period for a sample of 90 homes</td>
</tr>
<tr>
<td>21</td>
<td>Ahmad et al., [25]</td>
<td>Transmuted Inverse Rayleigh</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>23</td>
<td>Khan and King [27]</td>
<td>Transmuted Inverse Weibull</td>
<td>Survival times of 128 bladder cancer patients</td>
</tr>
<tr>
<td>24</td>
<td>Merovci et al., [28]</td>
<td>Transmuted Generalized Inverse Weibull</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>25</td>
<td>Khan and King [29]</td>
<td>Transmuted Generalized Inverse Weibull</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>26</td>
<td>Hussain [30]</td>
<td>Transmuted Gamma Exponintated</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>27</td>
<td>Elbatal et al., [31]</td>
<td>Transmuted Fréchet Exponintated</td>
<td>Winspeed (Carmeron Highland, Malaysia)</td>
</tr>
<tr>
<td>28</td>
<td>Ahmad et al., [32]</td>
<td>Transmuted Weibull</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>29</td>
<td>Lucena et al., [33]</td>
<td>Transmuted Generalized Gamma</td>
<td>Secondary Reactor Pumps</td>
</tr>
<tr>
<td>30</td>
<td>Owoloko et al., [34]</td>
<td>Transmuted Exponential</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>31</td>
<td>Abdul-Moniem [35]</td>
<td>Transmuted Burr Type III</td>
<td>Life of fatigue fracture of Kevlar 373/epoxy</td>
</tr>
<tr>
<td>33</td>
<td>Khan and King [37]</td>
<td>Transmuted Modified Inverse Rayleigh</td>
<td>30 successive values of March Precipitation (in inches)</td>
</tr>
</tbody>
</table>
| 34      | Mansour et al., [38] | Transmuted Additive Weibull | (i) Ages for 155 patients of breast tumors (Egypt)  
(ii) Failure time of 50 items reported in Aarset (1987) |
(ii) Strengths of 1.5cm glass fibres (England) |
<p>| 36      | Afify et al., [40] | Transmuted Weibull-Lomax | Gauge lengths of 10mm |
| 37      | Iriarte and Astorga [41] | Transmuted Generalized Rayleigh | Break life by fatigue of Kevlar 49/epoxy filaments |
| 38      | Granzotto and Louzada [42] | Transmuted Log-Logistic | Tabapua Race Cow data (Brazil) |</p>
<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Author(s) (Year)</th>
<th>Distribution</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>Luguterah and Nariru [44]</td>
<td>Transmuted Exponential Pareto</td>
<td>Fatigue life of 6061-T6 Aluminium coupons Data</td>
</tr>
<tr>
<td>41</td>
<td>Fatima and Roohi [45]</td>
<td>Transmuted Exponentiated-Pareto</td>
<td>(i) Breaking stress of carbon fibres (ii) Strengths of 1.5cm glass fibres (England)</td>
</tr>
<tr>
<td>42</td>
<td>Mansour and Mohamed [46]</td>
<td>Transmuted Lindley</td>
<td>128 bladder cancer patients</td>
</tr>
<tr>
<td>43</td>
<td>Khan et al., [47]</td>
<td>Transmuted Chen Lifetime</td>
<td>Strengths glass of fibres data.</td>
</tr>
<tr>
<td>44</td>
<td>Khan et al., [48]</td>
<td>Transmuted Kumaraswamy</td>
<td>(i) Flood data (ii) Infants born to HIV+ve women</td>
</tr>
<tr>
<td>45</td>
<td>Elgarhy et al., [49]</td>
<td>Transmuted Generalized Lindley</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>46</td>
<td>Vardhan and Balaswamy [50]</td>
<td>Transmuted Modified Weibull</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>47</td>
<td>Afify et al., [51]</td>
<td>Transmuted Weibull-Pareto</td>
<td>Gauge lengths of 10 mm</td>
</tr>
<tr>
<td>48</td>
<td>Shahzad and Asghar [52]</td>
<td>Transmuted Dagum</td>
<td>Rainfall data for the city of Islamabad, Pakistan</td>
</tr>
<tr>
<td>49</td>
<td>Khan et al., [53]</td>
<td>Transmuted Gompertz</td>
<td>Failure times of windshields data</td>
</tr>
<tr>
<td>50</td>
<td>Haq et al., [54]</td>
<td>Transmuted Power Function</td>
<td>(i) Strengths of 1.5 cm glass fibres (ii) Failure times of 50 items</td>
</tr>
<tr>
<td>51</td>
<td>Chakraborty and Bhati [55]</td>
<td>Transmuted Geometric</td>
<td>(i) A fleet of 13 Boeing 720 jet airplanes (ii) Vinyl chloride concentration (iii) Protein amount for adult patients (Chilean Hospital)</td>
</tr>
<tr>
<td>52</td>
<td>Bourguignon et al., [56]</td>
<td>Transmuted Birnbaum-Saunders</td>
<td>(i) Bladder cancer data (ii) ICU data</td>
</tr>
<tr>
<td>53</td>
<td>Khan et al., [57]</td>
<td>Transmuted Generalized Exponentiated Exponential</td>
<td>(i) Carbon fibre data (ii) Bladder cancer data</td>
</tr>
<tr>
<td>54</td>
<td>Cordeiro et al., [58]</td>
<td>Transmuted Modified Weibull</td>
<td>(i) Survival times of 72 guinea pigs infected with virulent tubercle bacilli</td>
</tr>
<tr>
<td>55</td>
<td>Elgarhy et al., [59]</td>
<td>Transmuted Generalized Quasi Lindley</td>
<td>(ii) March precipitation (in inches) in Minneapolis/St Paul</td>
</tr>
<tr>
<td>56</td>
<td>Khan et al., [60]</td>
<td>Transmuted Generalized Inverse Weibull</td>
<td>(i) Ball bearings data (ii) Fatigue life of aluminium data</td>
</tr>
</tbody>
</table>
| 57      | Khan et al., [61] | Transmuted Weibull | (i) Nicotine measurements made in several brands of cigarettes in 1995 (ii) Headache relief patient’s }
<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Author(s) (Year)</th>
<th>Distribution</th>
<th>Applications</th>
</tr>
</thead>
</table>
(ii) Greenwich data |
| 59 | Chhetri et al., [63] | Transmuted Kumaraswamy Pareto | (i) Exceedances of flood peaks of the Wheaton River  
(ii) Norwegian fire insurance data |
| 60 | Deka et al., [64] | Transmuted Exponentiated Gumbel | Water quality data using some water quality parameters |
| 61 | Al-Omari et al., [65] | Transmuted Janardan | Theoretical development without application |
(ii) Survival times of 121 patients with breast cancer |
| 63 | Venegas et al., [67] | Transmuted Exponentiated Maxwell | Single edge V-notched Aluminium plate repaired with Kevlar 49/epoxy |
| 64 | Nofal et al., [68] | Transmuted Exponentiated Additive Weibull | (i) Breaking stress of carbon fibres  
(ii) Nicotine measurements made in several brands of cigarettes |
(ii) Survival times of 72 guinea pigs  
(iii) 155 patients suffering from breast cancer |
| 66 | Arshad et al., [70] | Transmuted Exponentiated Moment Pareto | (i) Exceedances of flood peaks of the Wheaton River  
(ii) Remission times of bladder cancer 128 patients  
(iii) Kevlar 49/epoxy strands failure times  
(iv) Waiting time before the customer receives service in a bank |
| 67 | Khan et al., [71] | Transmuted Modified Weibull | Nicotine measurements made in several brands of cigarettes in 1995 |
| 68 | Abdullahi and Ieren [72] | Transmuted Exponential Lomax | Theoretical development without application |
| 69 | Elgarhy et al., [73] | Transmuted Kumaraswamy Quasi Lindley | (i) Strength data of glass of the aircraft window  
(ii) Relief times of 20 patients receiving an analgesic  
(iii) Waiting times before services of 100 bank customers |
<p>| 70 | Balaswamy [74] | Transmuted Half Normal | (i) March precipitation (in inches) in Minneapolis/St Paul |</p>
<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Author(s) (Year)</th>
<th>Distribution</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>71</td>
<td>Haq et al., [75]</td>
<td>Transmuted Weibull Power Function</td>
<td>(i) Strengths of 1.5 cm glass fibres (ii) Breaking stress of carbon fibres</td>
</tr>
<tr>
<td>72</td>
<td>Tahir et al., [76]</td>
<td>Transmuted New Weibull-Pareto</td>
<td>(i) Exceedances of flood peaks of the Wheaton River (ii) Floyd River flood rates for years 1935–1973</td>
</tr>
<tr>
<td>73</td>
<td>Khan [77]</td>
<td>Transmuted Generalized Inverted Exponential</td>
<td>Survival times for the 50 devices</td>
</tr>
<tr>
<td>75</td>
<td>Okorie and Akpanta [79]</td>
<td>Transmuted Generalized Inverted Expo.</td>
<td>50 devices put on life test at time zero</td>
</tr>
<tr>
<td>76</td>
<td>Abayomi [80]</td>
<td>Transmuted Half Normal</td>
<td>Buying behaviour data culled from a standard wholesale outlet</td>
</tr>
<tr>
<td>77</td>
<td>Khan et al., [81]</td>
<td>Transmuted Burr Type X</td>
<td>Fatigue fracture data and multiple myeloma patient’s data.</td>
</tr>
<tr>
<td>78</td>
<td>Gharaibeh and Al-Omari [82]</td>
<td>Transmuted Ishita</td>
<td>Fatigue fracture of kevlar 373/epoxy</td>
</tr>
<tr>
<td>80</td>
<td>Samuel [84]</td>
<td>Transmuted Logistic</td>
<td>March precipitation in Minneapolis/ St Paul.</td>
</tr>
<tr>
<td>81</td>
<td>Khan [85]</td>
<td>Transmuted Modified Inverse Weibull</td>
<td>Survival remission times of bladder cancer data</td>
</tr>
<tr>
<td>82</td>
<td>Khan et al., [86]</td>
<td>Transmuted Exponentiated Weibull</td>
<td>Failure times of 50 components (per 1000h) data</td>
</tr>
<tr>
<td>83</td>
<td>Ishaq et al., [87]</td>
<td>Transmuted Generalized Rayleigh</td>
<td>(i) Flood data sets with 20 observations (ii) Time of failure and running times for a sample of devices</td>
</tr>
<tr>
<td>84</td>
<td>Riffi et al., [88]</td>
<td>Generalized Fréchet Transmuted</td>
<td>Leukemia free-survival times for the 46 autologous transplant patients</td>
</tr>
<tr>
<td>85</td>
<td>Menezes et al., [89]</td>
<td>Transmuted Half-Normal</td>
<td>Daily series of daily precipitation</td>
</tr>
<tr>
<td>86</td>
<td>Aijaz et al., [90]</td>
<td>Transmuted Inverse Lindley</td>
<td>(i) Relief times of 20 patients getting an analgesic (ii) Breaking stress of carbon fibres</td>
</tr>
<tr>
<td>87</td>
<td>Yadav et al., [91]</td>
<td>Transmuted Lifetime</td>
<td>128 bladder cancer patients</td>
</tr>
</tbody>
</table>
of transmuted where are defined as follows in equation (4), (5) and (6) respectively. Moreover, we see that at And if let consider the following order. Let be independent and identically random variables distributed with distribution . Now consider the following order. 

\[
X_{1,3} = \min (X_1, X_2, X_3), \quad X_{2,3} = \text{the 2nd smallest of } (X_1, X_2, X_3) \quad \text{and} \quad X_{3,3} = \max (X_1, X_2, X_3)
\]

And let with probability \(\pi_1\),

\[
Y \equiv X_{1,3}^{\lambda_1}
\]

with probability \(\pi_2\),

\[
Y \equiv X_{2,3}^{\lambda_2}
\]

with probability \(\pi_3\),

Where \(\sum_{i=1}^{3}\pi_i = 1 \Rightarrow \pi_3 = 1 - \pi_1 - \pi_2\). Evidently \(F_T(x)\) is given by

\[
F_T(x) = \pi_1 \Pr(\min(X_1, X_2, X_3) \leq x) + \pi_2 \Pr(\min(X_1, X_2, X_3) \leq x) + \pi_3 \Pr(\min(X_1, X_2, X_3) \leq x) = 3\pi_1 G^2(x) + 3(\pi_2 - \pi_1) G^2(x) + (1 - 3\pi_2) G^3(x)
\]

And if \(3\pi_1 = \lambda_1\) and \(3\pi_2 = \lambda_2\), the above distribution becomes

\[
F(x) = \lambda_1 G(x) + (\lambda_2 - \lambda_1) [G(x)]^2 + (1 - \lambda_2) [G(x)]^3
\]

(3)

We see that at \(\lambda_1 = \lambda_2 = 1\), the above distribution gives the baseline distribution.

Moreover, Rahman et al., [94-96] have introduced three new cubic transmuted families of distributions which are defined as follows in equation (4), (5) and (6) respectively.

\[
F(x) = (1 + \lambda_1) G(x) + (\lambda_2 - \lambda_1) G^2(x) - \lambda_2 G^3(x), x \in R,
\]

(4)

where \(\lambda_1 \in [-1, 1]\) and \(\lambda_2 \in [-1, 1]\) and \(-2 \leq \lambda_1 + \lambda_2 \leq 1\). It can be easily observed that the cubic transmuted family of distributions proposed by AL-Kadim and Mohammed [97] turned out to be a special case of (4) for \(\lambda_2 = -\lambda_1\).

\[
F(x) = (1 + \lambda_1 + \lambda_2) G(x) - (\lambda_3 + 2\lambda_2) G^2(x) + \lambda_2 G^3(x), x \in R,
\]

(5)

where \(\lambda_1 \in [-1, 1]\) and \(\lambda_2 \in [0, 1]\).

2.2 Cubic rank transmutation map

Recently, a new family of transmutation map, named Cubic Rank Transmutation (CRT) is introduced by Granzotto et al., [2]. They developed the CRT log-logistic and CRT Weibull distributions which offer tractable distributions and are able to fit complex data sets such as ones with bimodal distribution or bimodal hazard rates.

Let be independent and identically random variables distributed with distribution \(G(x)\). Now consider the following order.

\[
X_{1,3} = \min (X_1, X_2, X_3), \quad X_{2,3} = \text{the 2nd smallest of } (X_1, X_2, X_3) \quad \text{and} \quad X_{3,3} = \max (X_1, X_2, X_3)
\]

And let with probability \(\pi_1\),

\[
Y \equiv X_{1,3}^{\lambda_1}
\]

with probability \(\pi_2\),

\[
Y \equiv X_{2,3}^{\lambda_2}
\]

with probability \(\pi_3\),

Where \(\sum_{i=1}^{3}\pi_i = 1 \Rightarrow \pi_3 = 1 - \pi_1 - \pi_2\). Evidently \(F_T(x)\) is given by

\[
F_T(x) = \pi_1 \Pr(\min(X_1, X_2, X_3) \leq x) + \pi_2 \Pr(\min(X_1, X_2, X_3) \leq x) + \pi_3 \Pr(\min(X_1, X_2, X_3) \leq x) = 3\pi_1 G^2(x) + 3(\pi_2 - \pi_1) G^2(x) + (1 - 3\pi_2) G^3(x)
\]

And if \(3\pi_1 = \lambda_1\) and \(3\pi_2 = \lambda_2\), the above distribution becomes

\[
F(x) = \lambda_1 G(x) + (\lambda_2 - \lambda_1) [G(x)]^2 + (1 - \lambda_2) [G(x)]^3
\]

(3)

We see that at \(\lambda_1 = \lambda_2 = 1\), the above distribution gives the baseline distribution.

Moreover, Rahman et al., [94-96] have introduced three new cubic transmuted families of distributions which are defined as follows in equation (4), (5) and (6) respectively.

\[
F(x) = (1 + \lambda_1) G(x) + (\lambda_2 - \lambda_1) G^2(x) - \lambda_2 G^3(x), x \in R,
\]

(4)

where \(\lambda_1 \in [-1, 1]\) and \(\lambda_2 \in [-1, 1]\) and \(-2 \leq \lambda_1 + \lambda_2 \leq 1\). It can be easily observed that the cubic transmuted family of distributions proposed by AL-Kadim and Mohammed [97] turned out to be a special case of (4) for \(\lambda_2 = -\lambda_1\).

\[
F(x) = (1 + \lambda_1 + \lambda_2) G(x) - (\lambda_3 + 2\lambda_2) G^2(x) + \lambda_2 G^3(x), x \in R,
\]

(5)

where \(\lambda_1 \in [-1, 1]\) and \(\lambda_2 \in [0, 1]\).
\[ F(x) = (1 - \lambda)G(x) + 3\lambda G^2(x) - 2\lambda G^3(x), \ x \in R, \]  \hspace{1cm} (6)

where \( \lambda \in [-1, 1] \).

Aslam et al., [98], introduced another cubic transmuted-G family of distributions and its related statistical properties. The lists of several cubic transmuted distributions introduced by various researchers are mentioned in Table 2.

**Table 2. Development in cubic rank transmuted distributions**

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>Authors (Year)</th>
<th>Distribution</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Granzotto et al. [2]</td>
<td>Cubic Transmuted Log-Logistic</td>
<td>Cattle sexual precocity data</td>
</tr>
<tr>
<td>3</td>
<td>AL-Kadim and Mohammed [97]</td>
<td>Cubic Transmuted Weibull</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>5</td>
<td>Rahman et al., [99]</td>
<td>Cubic Transmuted Pareto</td>
<td>(i) Life of fatigue fracture of Kevlar 373/epoxy (ii) Floyd River Dataset</td>
</tr>
<tr>
<td>6</td>
<td>Ansari and Eledum [100]</td>
<td>Cubic Transmuted Pareto</td>
<td>(i) Wheaton River Flood Peaks Data Set (ii) Floyd River Flood Data Set</td>
</tr>
<tr>
<td>7</td>
<td>Rahman et al., [95]</td>
<td>Cubic Transmuted Exponential</td>
<td>(i) The Wheaton River Data (ii) The Floyd River Data</td>
</tr>
<tr>
<td>8</td>
<td>Celik [101]</td>
<td>Cubic Transmuted Frêchet</td>
<td>Wind speed data</td>
</tr>
<tr>
<td>9</td>
<td>Celik [101]</td>
<td>Cubic Transmuted Gumbel</td>
<td>Water Quality Data</td>
</tr>
<tr>
<td>10</td>
<td>Celik [101]</td>
<td>Cubic Transmuted Gompertz</td>
<td>Failure Data</td>
</tr>
<tr>
<td>11</td>
<td>Saraçoğlu and Tanş [102]</td>
<td>Cubic Rank Transmuted Kumaraswamy</td>
<td>(i) Milk production data (ii) Operation and empirical data</td>
</tr>
<tr>
<td>12</td>
<td>Riffi and Hamdan [103]</td>
<td>Cubic Transmuted Gompertz-Makeham</td>
<td>Theoretical development without application</td>
</tr>
<tr>
<td>13</td>
<td>Ansari et al., [104]</td>
<td>Cubic Transmuted Power Function (CTPFD)</td>
<td>(i) 100 data points simulated from CTPFD (ii) 72-hour acute salinity tolerance of river marine invertebrates (iii) Failure times of 50 components</td>
</tr>
<tr>
<td>14</td>
<td>Rahman et al., [105]</td>
<td>Cubic Transmuted Weibull</td>
<td>(i) Carbon Fibres Data (ii) The Wheaton River Data</td>
</tr>
<tr>
<td>15</td>
<td>Bhatti et al., [106]</td>
<td>Cubic Transmuted Burr III-Pareto</td>
<td>(i) Tensile strength of carbon fibres (ii) Strengths of glass fibres</td>
</tr>
<tr>
<td>16</td>
<td>Rahman et al., [96]</td>
<td>Cubic Transmuted Uniform</td>
<td>Lifetimes of 30 electronic devices</td>
</tr>
<tr>
<td>17</td>
<td>Adeyinka [107]</td>
<td>Cubic Transmuted Exponentiated Exponential</td>
<td>Infant mortality rate per 1,000 live births in Nigeria</td>
</tr>
</tbody>
</table>
Generalized transmuted continuous distributions which Ali et al. distributions.

Utilizing the above equation (7) there is a huge scope to develop quartic rank transmuted probability distributions. Moreover, using the same concept we can generalize the $n^{th}$ rank mapped transmuted distributions.

Ali et al., [111] generated a new Generalized Rank Mapped Transmuted Distribution for generating families of continuous distributions which is defined as follows

Generalized transmuted cdf of $n^{th}$ rank mapped ($n = 1, 2, 3, \ldots$) distribution is given by

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>Authors (Year)</th>
<th>Distribution</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>Eledum [108]</td>
<td>Cubic Transmuted Exponentiated Pareto-I</td>
<td>(i) Failure times of Kevlar 49/epoxy strands (ii) Failure Times (in hours) of 50 Components</td>
</tr>
<tr>
<td>19</td>
<td>Ogunde et al., [109]</td>
<td>Cubic Transmuted Gompertz</td>
<td>Remission times of a random sample of 128 bladder cancer patients</td>
</tr>
<tr>
<td>20</td>
<td>Akter et al., [110]</td>
<td>Cubic Transmuted Burr-XII</td>
<td>(i) Life of fatigue fracture of Kevlar 373/ epoxy (ii) Remission times of a random sample of 128 bladder cancer patients</td>
</tr>
<tr>
<td>21</td>
<td>Ali et al., [111]</td>
<td>Cubic Transmuted Weibull</td>
<td>Theoretical development without application</td>
</tr>
</tbody>
</table>

2.3 Quartic Rank Transmutation Map

We can easily obtain the Quartic Transmuted Families of Distributions using the concept of Granzotto et al., [2]. Let $X_1, X_2, X_3$ and $X_4$ be independent and identically random variables distributed with distribution $G(x)$. Now consider the following order.

$$X_{1:4} = \min (X_1, X_2, X_3, X_4)$$

$$X_{2:4} = \min (X_1, X_2, X_3, X_4)$$

$$X_{3:4} = \min (X_1, X_2, X_3, X_4)$$

$$X_{4:4} = \max (X_1, X_2, X_3, X_4)$$

And let

$$Y \equiv X_{1:4}, \quad \text{with probability } \pi_1,$$

$$Y \equiv X_{2:4}, \quad \text{with probability } \pi_2,$$

$$Y \equiv X_{3:4}, \quad \text{with probability } \pi_3,$$

$$Y \equiv X_{4:4}, \quad \text{with probability } \pi_4,$$

Where $\sum_{i=1}^{4} \pi_i = 1 \Rightarrow \pi_4 = 1 - \pi_1 - \pi_2 - \pi_3$. Evidently $F_Y(x)$ is given by

$$F_Y(x) = \pi_1 \Pr(\min(X_1, X_2, X_3, X_4) \leq x) + \pi_2 \Pr(X_{2:4} \leq x) + \pi_3 \Pr(X_{3:4} \leq x) + \pi_4 \Pr(\max(X_1, X_2, X_3, X_4) \leq x)$$

$$= 2(2\pi_1)G(x) + 3(2\pi_2 - 2\pi_1)G^2(x) + 2(2\pi_3 - 4\pi_2 + 2\pi_3)G^3(x) + (1 - 2\pi_4 + 2\pi_2 - 4\pi_3)G^4(x)$$

And if $2\pi_1 = \lambda_1$ and $2\pi_2 = \lambda_2$, $2\pi_3 = \lambda_3$, the above distribution becomes

$$F(x) = 2\lambda_1 G(x) + 3(\lambda_2 - \lambda_1)G^2(x) + 2(\lambda_1 - 2\lambda_2 + \lambda_3)G^3(x) + (1 - \lambda_1 + \lambda_2 - 2\lambda_3)G^4(x) \quad (7)$$

At $\lambda_1 = \lambda_2 = \lambda_3 = \frac{1}{2}$ in (7), the above distribution becomes the baseline distribution.

Utilizing the above equation (7) there is a huge scope to develop quartic rank transmuted probability distributions. Moreover, using the same concept we can generalize the $n^{th}$ rank mapped transmuted distributions.

Imliyangba et al.; AJPAS, 13(3): 44-61, 2021; Article no.AJPAS.69356
where, \( I_{\alpha}(x) \) is incomplete beta function ratio and the corresponding generalized transmuted probability distribution function (pdf) of \( n^{th} \) rank mapped distribution is

\[
f_r(x) = g(x)[G(x)]^{n-1} \sum_{r=1}^{n} \sum_{j=0}^{n-r} k_{rj}(x)
\]

where \( g(x) \) is the pdf of a continuous population drawn from a random sample of size \( n \),

\[
k_i = \sum_{j=0}^{n-i} k_{ij}
\]

and

\[
k_{ij} = \frac{(-1)^{n-i}/n!}{B(i, n-i) \binom{n-i}{j}} \left[ G(x) \right]^{-j}
\]

### 3 Conclusion

Generalization of probability distributions through transmutation was first applied in the area of financial mathematics. As a result, several researchers have successfully applied this technique to model lifetime and survival data. At present, this approach is being applied in the areas of biology, engineering, environmental, medical among others to handle more complex (bi-modal) data.

In this work, we have reviewed the rank transmuted distributions which includes the quadratic and cubic rank transmuted distributions. We have also provided the quadratic and cubic rank transmuted distributions in table 1 and 2, along with their respective authors and applications. We expect that this review work will be of great value in the field of statistics.

As for the scope of future, Bivariate and multivariate transmuted rank distributions along with its properties can be studied. Also, Bayesian statistical inference which is one of the most important areas of research can be done for generalized rank mapped Transmuted distribution.

### Competing Interests

Authors have declared that no competing interests exist.

### References


[52] Shahzad MN, Asghar Z. Transmuted dagum distribution: A more flexible and broad shaped hazard


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