Valuation of Agricultural Commodity in an Unstructured Nigerian Market Using Black-Scholes’ Model

Onyegbuchulem Chialuka Adline1*, Nwobi Felix Noyanim2 and Onyegbuchulem Besta Okey3

1Department of Mathematics, Alvan Ikoku Federal College of Education, Owerri, Nigeria.
2Department of Statistics, Imo State University, Owerri, Nigeria.
3Department of Statistics, Imo State Polytechnic, Umuagwo, Nigeria.

Authors’ contributions

This work was carried out in collaboration among all authors. Author OCA designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors NFN and OBO edited the manuscript, managed the analysis of the study and literature searches. All authors read and approved the final manuscript.

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Abstract

This paper investigates the pricing accuracy of the Black-Scholes’ model in an unstructured over-the-counter Nigerian agricultural commodity market. Rice prices were valued using the Black-Scholes’ equation. The model’s predicted prices were compared with the market observed prices. Data for the study was sourced from food prices watch from January, 2017 - February, 2020 from National Bureau of Statistics. The result of the study shows that the prices produced by the Black-Scholes’ model provides a good match with the market observed prices. There is also a positive and significant correlation between the Black-Scholes’ model and market observed prices. Recommendation was made that Black-Scholes’ model should be used as an efficient model for pricing derivatives contracts of Nigerian local securities.

*Corresponding author: E-mail: ochialuka@yahoo.com;
**1 Introduction**

Commodity is an important kind of financial asset among others, especially under the process of globalization all over the world. The markets of commodity such as agricultural products like wheat, rice, cotton, palm oil, cocoa, coffee, rice etc. have a very wide and radical influence on the world’s economy. Federal Government of Nigeria has strengthened efforts concerning the expansion of the economy and reducing over-dependence on crude oil, a lot of funds have been directed towards the commodity market especially the agricultural commodity like rice. Rice is a key staple food consumed in Nigeria. Current policy initiatives target at prioritizing the agricultural sector and reducing dependency from international imports, adopting production and supply of agricultural inputs [1]. Rice is now planted on approximately 3.7 million hectares of land in Nigeria, covering 10.6% of the 35 million hectares of land for the crop growing, out of the entire arable land area of approximately 70 million hectares. 77% of the cultivated area of rice is rain-fed, of which 47% is lowland and 30% upland. The range of planted varieties is assorted and consist of both local (which includes Dias, Santana, Ashawa, Yarsawaba, and Yarkwuwa) and improved varieties of traditional African rice. Derivatives of rice have been traded over the counter OTC for decades in Nigeria without any formal structure yet market participants have clearly managed volatility in prices.

Commodities trading environment is a structured environment where commodities transaction takes place which affects the development of the commodities market, its activities and the exchanges directly or indirectly. It consists of number of elements, which are fundamental to the structural design of the market and dynamic to the smooth and proficient functioning of a commodity market or exchange. The exchanges serve as a platform for obtaining price information. These prices serve as a guide to investors in making investment decisions.

There is a commodity market platform in Nigeria called Nigerian Commodity Exchange (NCX) but it is scarcely functional thereby leaving commodities trading to be carried out in an unstructured market platform. Most of the new exchanges of developing countries are yet to meet expectations. Some studies have been made by different researchers to ascertain the pricing accuracy of the Black-Scholes’ model [2]. and [3] investigated popular derivatives models on Chinese and Indian commodity markets respectively. The volume of trade on NCX is insignificant compared to that on the physical market. Possibly, it is assumed that a reliable price investigation cannot be achieved from countries with the informal, unregulated, over the counter commodity (OTC) markets [4]. studied the Black-Scholes’ model and gave detail analysis of the assumptions of the model saying that the weakness of Black-Scholes is that it is based on simplistic assumptions such as constant volatility and a normal distribution function for the given asset return. This led to substantial discrepancies between market prices and prices calculated under the Black-Scholes’ model which are most evident in the observation of different implied volatilities base on the strike prices and maturities [5]. tested the efficiency of the Black-Scholes options model for suitability in determining the prices of palm-oil futures in Nigeria’s physical market. The study made use of primary sample data from an over-the-counter, palm-oil physical market in Nkwo-Nnewi, Anambra State, Nigeria. Their approach geared towards developing the model’s implied contract prices using the past prices of the commodity at the period of peak and dip seasons for ten years. The palm-oil futures prices were compared with the calculated unit profit and loss margins of the same historical periods and Pearson’s correlation Coefficient was also used to test their correlation. The study established that Black-Scholes Options model is appropriate to be employed as a base for pricing unstructured over-the-counter seasonal commodity contracts [6].in studying the distribution of changes in commodity future prices parametric and non-parametric test of normality resolved that agricultural futures markets, logarithmic price changes do not follow the normal distribution. He then recommended that in using econometric forecast and forecast accuracy models, there is need to consider seasonality of variance [7] in testing the effectiveness of the Black-Scholes’ model in pricing the S&P CNX Nifty options traded at India’s NSE by correlating between the historical volatility and the Black-Scholes’ model implied volatility, established a significant and positive correlation between the historical volatility and the implied volatilities. Further tests carried out led to his concluding that Black-Scholes’ model predicts more efficiently than historic volatility for stock options. [8] compared the predictive results
of the Black-Scholes’ model to several other models and empirical test shows that Black-Scholes’ prices still compared better than alternative implied volatility forecast approaches for agricultural commodities, although marginally. They recommended that price and yield (profit margin) correlations should be reviewed to measure implied volatility.

For an unstructured market to be investigated, it requires a well-known model that is generally accepted for determining the price of derivatives contracts. However, The Black-Scholes’ model developed by [9] is one of the most popular and widely used option pricing models, many of the techniques and option pricing models used in financial theory and practice are rooted in the ideas and methods presented by these men. [10]. Since most of the reviewed literature revealed that the Black-Scholes’ model has been tested in efficient markets with near perfect liquidity, hence there is need to know the efficiency of this model in an unstructured over –the-counter market especially for agricultural commodities market in developing countries like Nigeria. This study aims at investigating the efficiency of the Black-Scholes’ model as a pricing model for rice futures in Nigeria. Specifically, tends to value rice prices using the Black-Scholes’ equation and compare the model’s predicted prices with the market observed prices. Data for this study was sourced from [11].

2 Methodology

Price of a call option \( C(S,t) \) follows the Black-Scholes-Merton partial differential equation given by:

\[
\frac{\partial C}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS \frac{\partial C}{\partial S} - rC = 0 \tag{1}
\]

satisfying terminal condition

\[
C(S,T) = (S - K)^+ \tag{2}
\]

and boundary conditions

\[
C(0,t) = 0, \quad \text{for} \quad 0 \leq t \leq T \tag{3}
\]

and

\[
C(S,t) \to S, \quad \text{as} \quad S \to \infty \tag{4}
\]

The following formulas give Black-Scholes price of a European call at time \( t \)

\[
C(S,K,T) = Se^{-q(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2), \tag{5}
\]

where

\[
d_1 = \frac{\ln \left( \frac{S}{K} \right) + \left( r + \frac{\sigma^2}{2} \right)(T-t)}{\sigma \sqrt{T-t}},
\]

\[
d_2 = d_1 - \sigma \sqrt{T-t}
\]

and \( N(\cdot) \) is the standard normal cumulative distribution function,
The stock price follows the Geometric Brownian Motion. In the risk neutral world, it has the following dynamics:

\[ \frac{\partial S_t}{\partial t} = (r - q) S_t dt + \sigma S_t \partial W_t \]  

where

\( S_t \) is the stock price at time \( t \), \( t \) is current time, \( r \) is the risk free interest rate, \( q \) is the dividend yield, assumed to be constant, \( \sigma \) is the volatility of the asset’s price, \( W_t \) is a Brownian motion, \( N(,) \) is the cumulative distribution function of a standard normal variable, \( K \) is the strike or exercise price, \( (T - t) \) is the time to maturity. All these parameters are easily obtainable from a standard market except volatility parameter \( \sigma \). The Black-Scholes’ model was built upon the European style option, in which a European option gives its owner the right to purchase or sell an underlying asset for a given price (exercise price, or strike price) on the expiration date. This is applicable to most agricultural futures contracts like rice.

2.1 Estimation of parameters

The Black-Scholes’ model predicted prices were generated for each month using MATLAB code: \textit{blsprice (Price, Strike, Rate, Time, Volatility)} which computes option prices using a Black-Scholes’ model. Assume that the commodity pays no dividend. The parameters of Black-Scholes’ model are easily obtainable from a standard market except volatility parameter \( \sigma \). Since our chosen commodity is traded in an unstructured market in Nigeria, strike price, time to maturity and volatility do not exist but trading goes on continuously. Hence, these unavailable parameters need to be estimated in order to put them as inputs to the Black-Scholes equation.

2.1.1 Estimating the volatility parameter \( \sigma \)

Volatility is the most crucial of all option trading concepts. Volatility indicators provide traders with an estimate of how much movement a stock can be expected to make over a given time frame and also helps traders understand whether an option is cheap or expensive relative to the historical facts of the underlying instrument. It can be calculated through two approaches: historical volatility and implied volatility. Implied volatility of the Black-Scholes’ model is the value of the volatility that makes the Black-Scholes price match the market price. That means, the implied volatility comes from the market. The historical volatility is the one, which is derived from time series of past market prices. But it is not possible to know the future volatility of the stocks for trading and so traders use historical volatility to trade derivatives. Volatility is the standard deviation and can be determined by two methods, one is using the direct formula, which is given as:

\[ \sigma = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \mu)^2}{n - 1}} \]  

where,
\( \sigma \) = Standard Deviation, \( X \) = Monthly price change of the security, \( \mu \) = Mean or Average of the security, \( n \) = Number of values in the dataset. The second method is using MATLAB Simulink. In this paper, we have used the MATLAB Simulink approach as it gives the standard deviation directly in its data statistics.

2.1.2 Estimating the strike price parameter \( (K) \)

Strike price is the fixed price of an option at which the owner of an option can buy or sell an underlying security. Strike price is one of the most important factors when it comes to value an option because without the strike price it is not possible to determine whether the option is valuable or worthless. Recall that the rate of growth from stock price to the strike price is given as

\[
X = \ln\left(\frac{K}{S}\right), \quad \text{then} \quad e^X = \left(\frac{K}{S}\right),
\]

so we can write strike price input in Black-Scholes equation as:

\[
K = \left(S \ast e^X\right).
\]

Following [12], we can find the monthly price change \( (X) \), which is for a current month, the natural log of current months price of the stock divided by last month’s price of the stock for a particular stock. So,

\[
X = \ln\left(\frac{\text{Current Month's Price}}{\text{Last Month's Price}}\right)
\]

Then averaging all monthly price changes of a particular year, we get the mean which is kept constant by changing all other variables and thus call option is calculated.

2.1.3 Estimating the time parameter \( (T - t) \)

\( (T - t) \) is the time to maturity, it is important to note that, the time is in years (theoretically) but option traders also work with days remaining expiration. There is no time of expiration in an unstructured market, people do trading unsystematically and uninterruptedly, which actually is a never ending process. So the time is taken to be the exact number of trading date on which the commodity will be traded over the total number of trading days of the year. However, the denominator side is kept constant but the numerator will change. So in this way by changing the time continuously along with stock price of any month, the value of options is determined.

2.1.4 Estimating the interest rate parameter \( (r) \)

Black-Scholes’ model assumes that the interest rate is known and constant through time and uses the risk-free rate to represent this constant and known rate. Though, [13] argued that in the real world, there is no such thing as a risk-free rate. The Nigeria government treasury bills short term rates was considered to be the risk free interest rates used in this paper. However, the value of risk free rate of interest was taken at 5.3%.

2.2 Correlation coefficient of Black-Scholes’ and market prices

The Karl Pearson Product Moment Correlation Coefficient was used to ascertain the correlation between the prices produced by Black-Scholes’ model and market traded prices between January, 2017 and February, 2020. The correlation coefficient is given as:
where, $r$ is the correlation coefficient which lies between -1 and +1, n is the number of periods, X is the variable that represents Black-Scholes’ model prices, Y is the variable that represents the market traded prices.

3 Results and Discussion

Table 1 shows the estimated strike prices ($K$), monthly price change of the security ($X$), the Black-Scholes’ model predicted prices and its comparison with the market prices in naira per kilogram.

Table 1. Comparison of the Black-Scholes’ model predicted and the market prices

<table>
<thead>
<tr>
<th>S/N</th>
<th>Month</th>
<th>Mrk. Price (₦)</th>
<th>X (₦)</th>
<th>K (₦)</th>
<th>B-S Price (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Jan.2017</td>
<td>286.1900</td>
<td>0.0000</td>
<td>286.1900</td>
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<tr>
<td>2</td>
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<td>327.8094</td>
<td>305.8521</td>
</tr>
<tr>
<td>3</td>
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<td>0.0084</td>
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<td>4</td>
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<td>290.0190</td>
<td>298.8928</td>
</tr>
<tr>
<td>5</td>
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<td>323.8200</td>
<td>0.0787</td>
<td>350.3343</td>
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<tr>
<td>6</td>
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<td>0.0052</td>
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<tr>
<td>7</td>
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<td>9</td>
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<td>11</td>
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<td>12</td>
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<tr>
<td>13</td>
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<td>14</td>
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<td>15</td>
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<td>18</td>
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<td>268.2257</td>
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<table>
<thead>
<tr>
<th>S/N</th>
<th>Month</th>
<th>Mrk. Price (₦)</th>
<th>X (₦)</th>
<th>K (₦)</th>
<th>B-S Price (₦)</th>
</tr>
</thead>
<tbody>
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<td>33</td>
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<td>0.0618</td>
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</table>

As Table 1 shows, the Black-Scholes’ model provides a good match with the market observed prices. Also, the correlation coefficient of Black-Scholes and market prices of rice in naira per kilogram traded between January,2017 and February,2020 is positive and significant at the 0.01 level (2-tailed).

4 Conclusion

The inspection of Tables 1 and 2 in this study shows that the prices produced by the Black-Scholes’ model provides a good match with the market observed prices in the data sample. This makes the Black-Scholes’ model usable as pricing tool in Nigeria’s unstructured market. Our result agrees with the report of [5] who stated that Black-Scholes options model is suitable to be used as a base for pricing unstructured over-the-counter seasonal commodity contracts.

It is therefore recommended that Black-Scholes’ model should be used as an efficient model for pricing derivatives contracts of Nigerian local securities. Nigerian government should also, corroborate with the organized private sectors to ensure the establishment of an organized derivatives exchange in Nigeria with the help of pricing tools like the Black-Scholes’ model because Nigeria’s economic growth is influenced and depends so greatly on developing a profound financial market, together with an active derivatives market where most of the natural resources of the country can be traded universally in a competitive and apparent manner. However further studies are still required to be done so as to improve the model’s efficiency by treating volatility as stochastic-process rather than constant considering the dynamic nature of agricultural commodity derivatives.

Competing Interests

Authors have declared that no competing interests exist.

References


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